

# Low-energy spin dynamics in the $[\text{YPc}_2]^0 S = \frac{1}{2}$ antiferromagnetic chain

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<sup>1</sup>H nuclear magnetic resonance (NMR) measurements in  $[\text{YPc}_2]^0$ , an organic compound formed by radicals stacking along chains, are presented. The temperature dependence of the macroscopic susceptibility of the NMR shift and of the spin-lattice relaxation rate  $1/T_1$  indicate that the unpaired electron spins are not delocalized but rather form a  $S = 1/2$  antiferromagnetic chain. The exchange couplings estimated from those measurements are all in quantitative agreement. The low-energy spin dynamics can be described in terms of diffusive processes and the temperature dependence of the corresponding diffusion constant suggests that a spin gap at  $\sim 1$  K might be present in this compound.

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## I. INTRODUCTION

Molecular solids have attracted much interest for decades owing to the possibility to easily tune their properties either through a chemical bottom up approach or by varying physical parameters as the external pressure.<sup>1</sup> One of the most versatile families of molecular solids is the one based on phthalocyanine ( $\text{Pc} = \text{C}_{32}\text{H}_{16}\text{N}_8$ ) molecules.<sup>2</sup> In fact, the employment of these materials in different areas, ranging from the fabrication of organic light-emitting diodes to contrast agents or spintronics materials, has been envisaged. Pc-based compounds have attracted significant interest in the past decade after it has been suggested that high-temperature superconductivity could be induced in these materials by alkali doping<sup>3</sup> and, more recently, when it has been recognized that neutral  $[\text{LnPc}_2]^0$  molecules, with Ln a lanthanide ion, are molecular nanomagnets with extremely long coherence times at liquid-nitrogen temperature.<sup>4–8</sup> Owing to the flat shape of Pc molecules, the structure of Pc-based materials is typically characterized by chains along which Pc molecules tend to stack.<sup>9</sup> Accordingly, some of the Pc-based materials show many similarities to the Beechgaard salts.<sup>10</sup>

The bis(phthalocyaninato) yttrium  $[\text{YPc}_2]^0$  compound can be considered to be the parent compound of the aforementioned  $[\text{LnPc}_2]^0$  molecular magnets. In fact, it is characterized by the absence of localized  $f$  electrons and the microscopic properties are mainly associated with the presence of an unpaired electron delocalized in the  $a_2\pi$  orbital, due to the one-electron oxidation of the  $[\text{YPc}_2]^-$  unit.<sup>11</sup> Thus,  $[\text{YPc}_2]^0$  allows to investigate the spin dynamics associated only with this unpaired electron spin, independently from the one due to  $f$  electrons. One of the most suitable tools to address this aspect is the nuclear magnetic resonance (NMR) technique. In this work we present an experimental study of the magnetic properties of a  $[\text{YPc}_2]^0$  compound by means of magnetization and NMR measurements. The temperature dependence of the macroscopic susceptibility, of the NMR shift, and of the spin-lattice relaxation rate  $1/T_1$  clearly show that this system is a prototype of a  $S = 1/2$  antiferromagnetic chain, characterized by a diffusive low-frequency spin dynamics and, possibly, by the presence of a low-energy spin gap.

## II. EXPERIMENTAL RESULTS AND DISCUSSION

$[\text{YPc}_2]^0$  polycrystalline samples were synthesized by using some modifications of the protocol published in Ref. 12. All reagents were purchased from Acros or Aldrich and used without further purification. A mixture of 1,2-dicyanobenzene (42 mmol),  $\text{Y}(\text{acac})_3 \cdot 4\text{H}_2\text{O}$  (5 mmol), and 1,8-diazabicyclo[5,4,0]undec-7-ene (DBU) (21 mmol) in 50 ml of 1-pentanol was refluxed for 1.5 days. The solution was allowed to cool to room temperature. The precipitate was collected by filtration and washed with *n*-hexane and  $\text{Et}_2\text{O}$ . The crude purple product was redissolved in 800 ml of  $\text{CHCl}_3/\text{MeOH}$  (1/1) and undissolved  $\text{PcH}_2$  was filtered off. Both forms, blue (anionic  $[\text{YPc}_2]^-$ ) and green (neutral  $[\text{YPc}_2]^0$ ), were obtained simultaneously, as revealed by electronic absorption spectra. In order to convert the unstabilized anionic form to the neutral one, the reaction mixture was presorbed on active ( $\text{H}_2\text{O}$ -0%) basic alumina oxide. Purification was carried out by column chromatography on basic alumina oxide (deactivated with 4.6%  $\text{H}_2\text{O}$ , level IV) with a chloroform-methanol mixture (10:1) as the eluent. In general, the yield was 30%–35%. According to a microelemental analysis based on atomic spectroscopic methods [inductively coupled plasma inductively coupled plasma (ICP)] performed at *Mikroanalytisches Labor Pascher*, the powder sample contains molecules of  $[\text{YPc}_2]^0$ , water, and  $\text{CH}_2\text{Cl}_2$  in a ratio of 1:1:1/3. The molecules crystallized in the space group  $P2_12_12_1$  ( $\gamma$  phase), as reported in Ref. 13.

dc magnetization ( $M$ ) measurements have been performed by using an MPMS-XL7 Quantum Design superconducting quantum interference device (SQUID) magnetometer. The magnetization was found to depend linearly on the magnetic-field intensity  $H$ , for  $H \leq 5$  kG, over the entire explored temperature range and, accordingly, the macroscopic static uniform susceptibility can be written as  $\chi_S = M/H$ . The temperature dependence of  $\chi_S$  reveals the presence of antiferromagnetic correlations. In fact,  $\chi_S(T)$  can be well reproduced by a Curie-Weiss (CW) law

$$\chi_S(T) = \frac{C}{T + \Theta} + \chi_0, \quad (1)$$

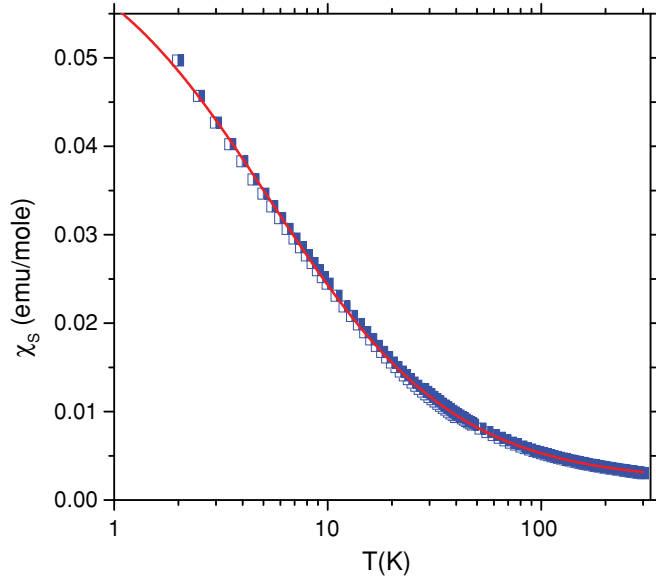


FIG. 1. (Color online) Temperature dependence of static uniform susceptibility  $\chi_S$  for a  $[\text{YPC}_2]^0$  complex, derived from SQUID magnetization measurements. The solid line shows the best fit of the data to Curie-Weiss law.

where  $C = g^2 \mu_B^2 S(S+1) N_A / (3k_B)$  is Curie constant ( $\mu_B$  the Bohr magneton,  $g$  the Landé factor,  $N_A$  Avogadro's number, and  $k_B$  the Boltzmann constant).  $\Theta$  is the CW temperature and  $\chi_0$  a temperature-independent term mainly due to diamagnetic and Van Vleck corrections. By fitting the data, leaving all three parameters free, we found an antiferromagnetic CW temperature  $\Theta = 5.37 \pm 0.04$  K (Fig. 1) and a Curie constant  $C = 0.342 \pm 0.002$  erg K/G<sup>2</sup>, quite close to the value 0.375 erg K/G<sup>2</sup>, which is expected for a  $S = 1/2$  system. If we fixed  $C = 0.375$  erg K/G<sup>2</sup> the fit was still good and the CW temperature was  $\Theta = 6.18 \pm 0.03$  K. The temperature dependence of  $\chi_S$  shows that the unpaired electron spins are localized along the chains formed by  $[\text{YPC}_2]^0$  molecules, even if a certain overlap of the  $\pi$  orbitals of adjacent molecules must be present in order to justify the magnitude of the antiferromagnetic exchange coupling  $J_e$ . In fact, although this system should present a narrow half-filled band, the sizable Hubbard Coulomb repulsion  $U \sim 1$  eV prevents the electron delocalization along the chain.<sup>14</sup> In this limit,  $J_e = \Theta = 4t^2/U$ , with  $t$  the hopping integral among adjacent molecules. From the estimated value of  $\Theta$ , one would derive a bandwidth, formed by the overlap of  $a_2$  orbitals in adjacent molecules,  $W = 4t \sim 0.05$  eV  $\ll U$ , justifying the spin localization along the chain.

The <sup>1</sup>H NMR spectra were obtained in the 1.6–300 K temperature range for magnetic-field intensities  $H = 9$ , 1, and 0.3 T. The spectra were derived from the Fourier transform of half of the echo formed after a  $\pi/2$ - $\tau$ - $\pi$  pulse sequence, when the full NMR line could be irradiated or, otherwise, from the envelope of the echo amplitude upon varying the irradiation frequency (Fig. 2). The line shape was Gaussian in the entire investigated temperature range. For  $H = 9$  and 1 T a broadening of the spectrum can be observed at low temperature, which is more pronounced at higher-field intensities (Fig. 3). On the other hand, for  $H = 0.3$  T the

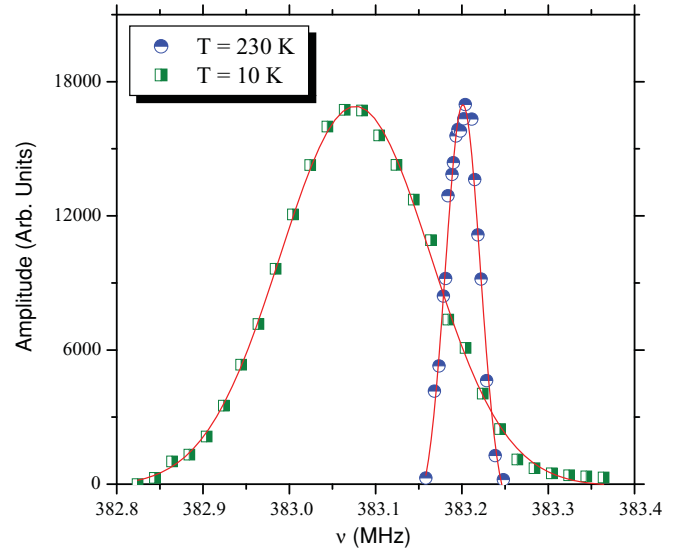


FIG. 2. (Color online) <sup>1</sup>H NMR spectra in  $[\text{YPC}_2]^0$ , for  $H = 9$  T, obtained from the envelope of the echo amplitude upon varying the irradiation frequency. A clear shift and broadening of the line is observed on cooling. The solid lines show the Gaussian best fits.

linewidth  $\Delta\nu$  is practically temperature independent and the broadening is likely to be due just to nuclear dipole-dipole interaction. The increase of the linewidth with  $H$  suggests that the low-temperature line broadening originates from some anisotropy in the hyperfine coupling, which for a powder gives rise to a linewidth proportional to the susceptibility. In fact, it is noticed that if we subtract the  $T$ -independent contribution at  $H = 0.3$  T from the raw data and divide the linewidth by the Larmor frequency  $\nu_0$ , the data at different fields overlap (see inset to Fig. 3). This result also indicates that there is no additional internal field due to the onset of a long-range magnetic order down to 1.6 K, demonstrating that the interchain coupling  $J_\perp \ll J_e$ .

The NMR paramagnetic shift  $\Delta K = (\nu_R - \nu_0)/\nu_0$ , with  $\nu_R$  the resonance frequency, shows a more pronounced temperature dependence (Fig. 4). As expected, it was found to increase upon cooling according to

$$\Delta K = \frac{A\chi_S}{2\mu_B N_A}, \quad (2)$$

namely the temperature dependence of  $\Delta K$  should be the same of the macroscopic spin susceptibility. In fact, also  $\Delta K(T)$  is found to obey a CW with a CW temperature  $\Theta = 7.4 \pm 0.3$  K, close to the one derived from SQUID magnetization measurements. The small difference between those two types of measurements could be associated with a tiny amount of impurities which might affect the macroscopic susceptibility. Accordingly, the measurement of the microscopic susceptibility with paramagnetic shift measurements is expected to provide a more reliable estimate of the static uniform spin susceptibility and of the CW temperature. By plotting  $\Delta K$  as a function of  $\chi_S$  a linear trend is attained (Fig. 4) and from the slope it is possible to estimate the isotropic term of the hyperfine coupling tensor  $A = 180 \pm 10$  G.

The <sup>1</sup>H nuclear spin-lattice relaxation rate  $1/T_1$  was measured in the 1.6–300 K temperature range and for different values of the external field.  $1/T_1$  was extracted from the

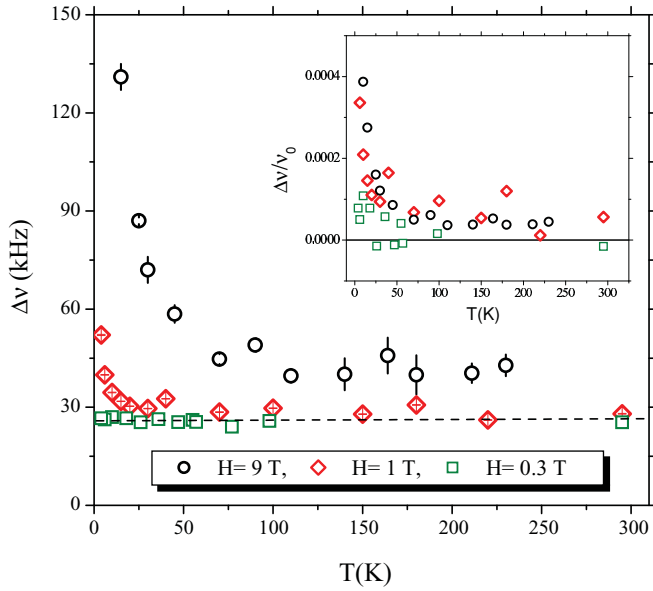


FIG. 3. (Color online) Temperature dependence of <sup>1</sup>H full NMR linewidth at half intensity in [YPC<sub>2</sub>]<sup>0</sup>, at three different magnetic fields. In the inset the linewidth is normalized by the Larmor frequency after subtracting the constant linewidth due to nuclear dipole-dipole interaction.

recovery of nuclear magnetization after a saturation recovery pulse sequence. The recovery law was found to be a single exponential in all the explored temperature range (see the inset to Fig. 5). This result is evidence that the unpaired electron is delocalized onto a  $\pi$  orbital within the molecule. In fact, since in the two phthalocyanine rings a large number of inequivalent proton sites is present, if the electron was on a more localized orbital, a distribution of hyperfine couplings would be present and, accordingly, a stretched exponential recovery law should

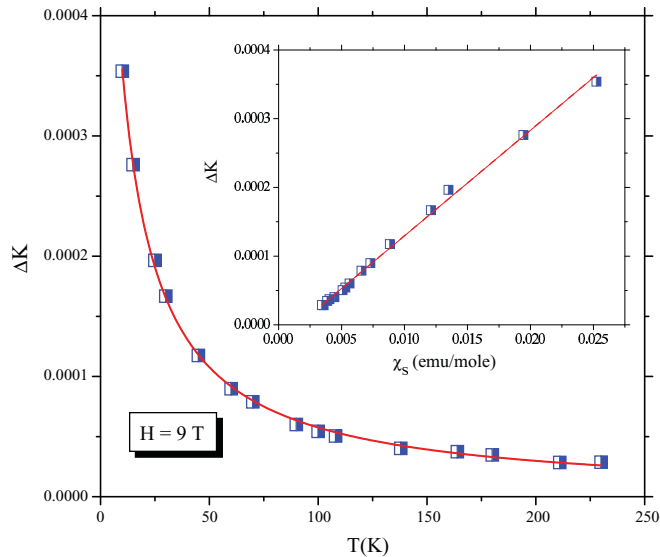


FIG. 4. (Color online) Temperature dependence of <sup>1</sup>H paramagnetic shift  $\Delta K$  in [YPC<sub>2</sub>]<sup>0</sup>. The solid line shows the best fit of the data to CW law. In the inset  $\Delta K$  is reported as a function of the macroscopic susceptibility. The solid line is the best fit according to a linear dependence of  $\Delta K$  vs  $\chi_S$ .

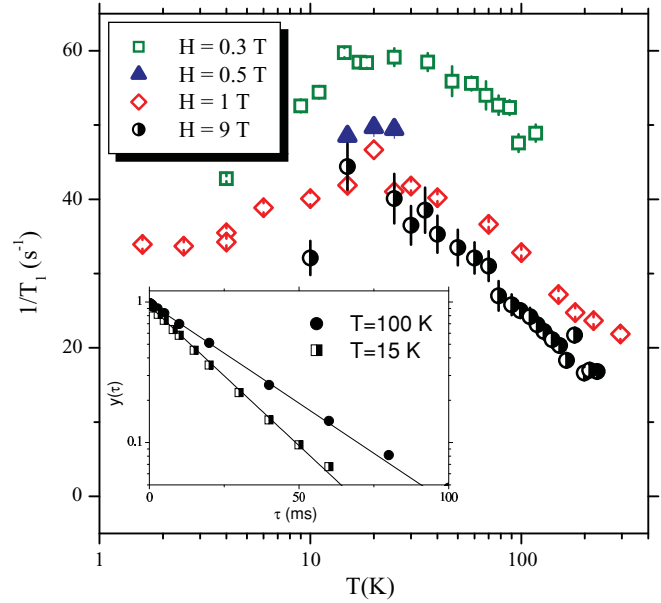


FIG. 5. (Color online) <sup>1</sup>H nuclear spin-lattice relaxation rate temperature dependence for a [YPC<sub>2</sub>]<sup>0</sup> compound measured for different values of the applied field. In the inset the recovery of the nuclear magnetization as a function of the delay  $\tau$  between the saturating and the echo readout sequences is shown at two different temperatures. The solid lines show the best fit for a single exponential recovery.

be observed. Moreover, the fact that hyperfine coupling seems basically isotropic indicates that it could originate from the contact interaction between the unpaired electron spin in the  $a_2 \pi$  orbital and the <sup>1</sup>H nuclei.

The temperature dependence of  $1/T_1$  at different magnetic fields is shown in Fig. 5. In general, for a relaxation process driven by electron-spin fluctuations one can write

$$\frac{1}{T_1} = \frac{\gamma^2}{2N} \sum_{\alpha, \mathbf{q}} [|A_{\mathbf{q}}|^2 S_{\alpha, \alpha}(\mathbf{q}, \omega_L)]_{\perp}, \quad (3)$$

where  $\gamma$  is the nuclear gyromagnetic ratio,  $|A_{\mathbf{q}}|^2$  is the form factor describing the hyperfine coupling with spin excitations at wave vector  $\mathbf{q}$ , and  $S_{\alpha, \alpha}(\mathbf{q}, \omega_L)$  ( $\alpha = x, y, z$ ) is the component of the dynamical structure factor at the Larmor frequency. In the high-temperature limit, namely, when the thermal energy is much larger than the exchange energy ( $T \gg \Theta$ ), the  $1/T_1$  of a spin  $S = 1/2$  antiferromagnet becomes temperature independent and is given by<sup>15</sup>

$$\frac{1}{T_1} = \frac{\gamma^2}{2} (A_x^2 + A_y^2) \frac{S(S+1) \sqrt{2\pi}}{3 \omega_H}, \quad (4)$$

where  $A_x \simeq A_y \simeq A$  are the components of the hyperfine coupling tensor which is basically isotropic, while  $\omega_H = (J_e k_B / \hbar) \sqrt{2zS(S+1)/3}$  is the Heisenberg exchange frequency, with  $z = 2$  the number of nearest-neighbor spins along the chain. By taking the measured value of  $1/T_1 \simeq 20 \text{ s}^{-1}$  at high temperature, from Eq. (4) it is possible to estimate an exchange frequency  $\omega_H \simeq 9.2 \times 10^{11} \text{ rad/s}$ , corresponding to an exchange coupling constant  $J_e \simeq 7.0 \text{ K}$ , in quite good agreement with the value which can be estimated from the NMR shift measurements. Upon decreasing the temperature,

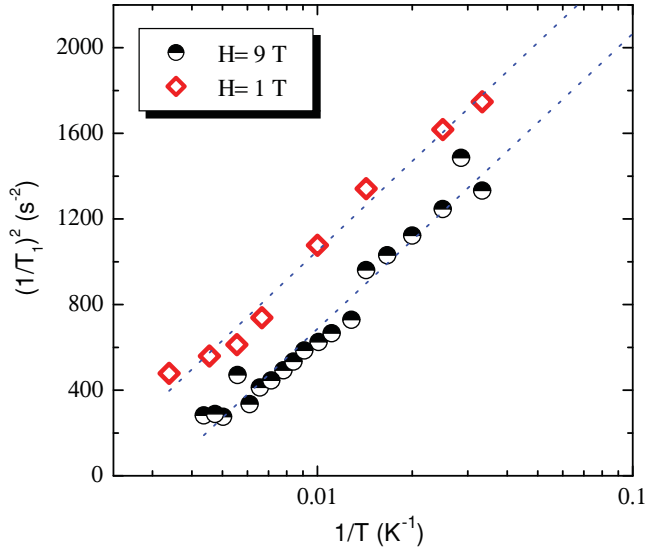


FIG. 6. (Color online)  $1/T_1$  squared is plotted as a function of  $1/T$ , in logarithmic scale, for two values of the external field [ $H = 9$  T (circles) and  $H = 1$  T (diamonds)]. The dashed lines represent the best fits to Eq. (5).

for  $200 \text{ K} \geq T \geq 30 \text{ K}$ , one observes a progressive slow increase of  $1/T_1$  (Fig. 5). In particular, it is noticed that the nuclear spin-lattice relaxation rate increases on decreasing temperature according to

$$1/T_1 \propto \ln^{1/2}(T_0/T). \quad (5)$$

In fact, in Fig. 6 one observes that  $(1/T_1)^2$  is a linear function of  $1/T$ , when reported in logarithmic scale. Remarkably, this logarithmic increase of  $1/T_1$  is expected in a  $S = 1/2$  Heisenberg antiferromagnet, but only for  $T \leq J_e$ .<sup>16</sup> Here it is not clear why the logarithmic behavior extends up to  $T \gg J_e$ .

At  $\sim 20$  K a peak in the nuclear spin-lattice relaxation rate appears (Fig. 5), whose intensity decreases by increasing the external field intensity. Eventually, below  $T \simeq 5$  K, the  $1/T_1$  is only weakly temperature dependent. The maximum in  $1/T_1$ , not associated with molecular motions, could be due to a form factor, which partially filters out the antiferromagnetic fluctuations as the system gets more and more correlated.

The magnetic field dependence of  $1/T_1$  (Fig. 7) can originate from the diffusive nature of the spin-correlation function, which in one dimension is characterized by long-time tails yielding to a divergence of the low-frequency spectral density  $J(\omega)$ .<sup>17</sup> In fact, in the presence of diffusive processes, for the spin excitations  $1/T_1$  can be written in terms of the spectral density for the spin excitations according to the following equation:<sup>18</sup>

$$\frac{1}{T_1} = \frac{\gamma^2 k_B T \chi_0}{2 (g\mu_B)^2} \left[ \frac{3}{5} A_d^2 J(\omega_0) + \left( A^2 + \frac{7}{5} A_d^2 \right) J(\omega_e \pm \omega_0) \right], \quad (6)$$

where  $A_d$  is the anisotropic term of the hyperfine coupling, which hereafter shall be neglected since  $A^2 \gg A_d^2$ . Then just the second term in square brackets can be considered. In Eq. (6)  $\chi_0$  is the static uniform susceptibility per spin and

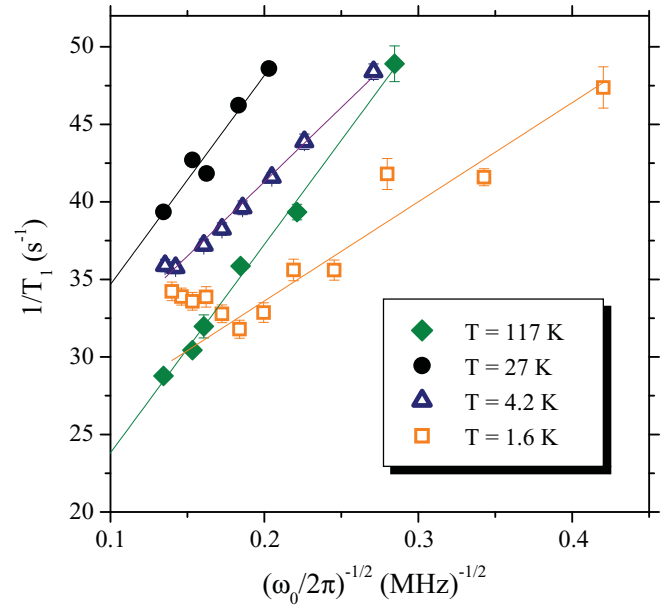


FIG. 7. (Color online) The  $^1\text{H}$  spin-lattice relaxation time  $1/T_1$  in  $[\text{YPC}_2]^0$  is plotted as a function of  $(\omega_0/2\pi)^{-1/2}$  for different selected temperatures. The solid lines show the best fit according to Eqs. (6) and (7) in the text.

$\omega_e = \omega_0 \gamma_e / \gamma$  is the electron resonance frequency. This means that during the nuclear relaxation process a simultaneous flip of the electron and nuclear spins occur, involving an energy exchange  $\hbar(\omega_e \pm \omega_0)$ , and thus  $1/T_1$  probes the spin excitations at a frequency close to  $\omega_e$ .

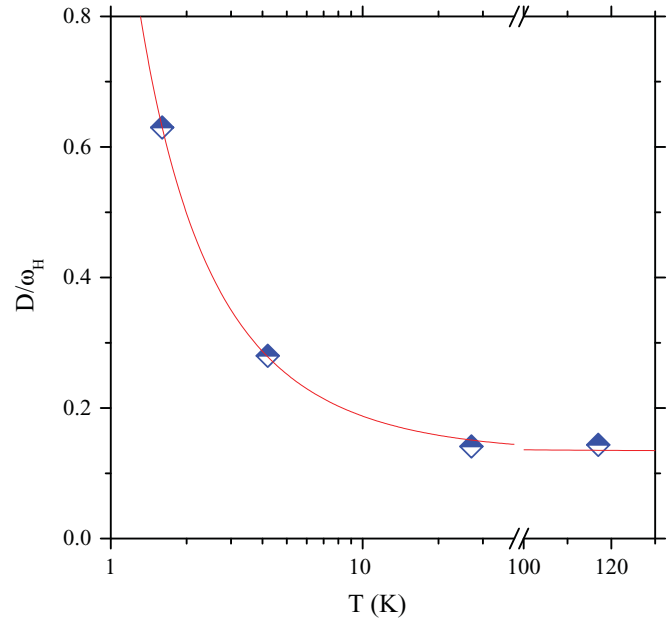


FIG. 8. (Color online) Temperature dependence of the ratio  $D/\omega_H$  between the spin-diffusion coefficient  $D$  and the exchange frequency  $\omega_H$  in a  $[\text{YPC}_2]^0$  compound as derived from the slopes in the  $1/T_1$  vs  $(\omega_0/2\pi)^{-1/2}$  plots in Fig. 7. The solid line gives the best fit according to  $D \propto \exp(\Delta/T)$  with  $\Delta = 1.2 \pm 0.4$  K.

In a one-dimensional system, the spectral density at  $\omega_e$  is characterized by a low-frequency divergence given by<sup>19</sup>

$$J(\omega_e) = \frac{1}{\sqrt{2D}} \left( \frac{\omega_c + \sqrt{\omega_e^2 + \omega_c^2}}{\omega_e^2 + \omega_c^2} \right)^{1/2}, \quad (7)$$

where  $\omega_c$  is a low-frequency cutoff accounting for the finite spin anisotropy and/or interchain coupling, while  $D$  is the spin-diffusion rate. In Fig. 7,  $1/T_1$  is plotted as a function of  $\nu_0^{-1/2}$ . The observed linear trend further proves the one-dimensional nature of the antiferromagnetic correlations. Moreover, the absence of a low-frequency flattening in the  $1/T_1$  plot indicates that spin diffusion occurs in the electronic frequency range  $\omega_c \ll \omega_e \ll D$ . Thus, from the slopes of the curves it is possible to deduce the spin-diffusion coefficient at different temperatures (Fig. 8), considering  $A \simeq 180$  G and neglecting  $\omega_c \ll \omega_e$  in Eqs. (6) and (7). The estimated spin-diffusion coefficient is of the order of the exchange frequency  $\omega_H$  and it is found to progressively decrease with temperature and to become nearly constant above 20 K. It is interesting to observe that  $D \propto \exp(\Delta/T)$ , namely, the behavior expected for one-dimensional antiferromagnets in the presence of a spin gap  $\Delta$  between singlet and triplet excitations.<sup>20</sup> Here we find that  $\Delta = 1.2 \pm 0.4$  K, suggesting that a small gap, either due to

competing exchange interactions or to a dimerization, might be present in [YPC<sub>2</sub>]<sup>0</sup>. It is interesting to observe that, at low temperature, when the Zeeman energy  $\hbar\omega_e \simeq \Delta$  the breakdown of Eq. (7) is noticed. In fact, in Fig. 7 one clearly notices that at  $T = 1.6$  K the linear behavior is no longer obeyed at high fields (i.e., low values for  $\sqrt{2\pi/\omega_0}$ ) and  $1/T_1$  ceases to decrease with increasing field. This could be due to the modifications in the spin correlations induced by the magnetic field for  $\hbar\omega_e \simeq \Delta$ , possibly associated with the progressive closure of the spin gap.

In conclusion, from magnetization, <sup>1</sup>H NMR paramagnetic shifts, and  $T_1$  measurements, we have derived the magnitude of the antiferromagnetic exchange interaction in a [YPC<sub>2</sub>]<sup>0</sup> compound and found an overall quantitative agreement among the different techniques. The low-energy spin excitations are of diffusive character and characteristic of one-dimensional antiferromagnets. From the temperature dependence of the spin-diffusion rate derived from  $1/T_1$  vs  $H$  measurements it was found that a spin gap at  $\sim 1$  K might be present in this compound.

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